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-Formulated Wavelet Spectral element for a healthy composite plates and used the formulated spectral element to obtain all the lamb wave modes. Validated the element with the conventional finite elements
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-Development Wavelet Spectral elements for a composite plates with arbitrarily oriented cracks
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Wavelet Spectral Finite Elements for Wave Propagation in Composite Plates- Years 3-4

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Wavelet Spectral Finite Elements for Wave Propagation in Composite Plates
3rd and 4th year technical progress report

(Contract no: FA23861214005)

Executive Summary

The principal goal of this research is to advance the technology of structural health management for composite structures. The use of composites for aerospace structures is increasing rapidly; however, composite structures are susceptible to impact damage and delaminations and cracks may reach critical length before visual detection. Wave propagation based methods have shown promise for SHM of composite structures. The spectral finite elements (SFE) method is highly suitable for wave propagation analysis due to its frequency domain approach, which yields models that are many orders smaller than conventional FEM. Also, the frequency domain formulation of WSFE enables direct relationship between output and input through system transfer function. Wavelet spectral finite element (WSFE) method overcomes the signal "wrap around" problem to accurately model 2-D plate structures of finite dimensions, unlike the existing Fourier transform based SFE. In addition, initial conditions can be elegantly specified. The specific accomplishments of the research are:

1. Developed WSFE model for Healthy plate, damaged composite plate with transverse crack and healthy stiffened structures (with plate beam connections and plate-plate connections);
2. Validated WSFE modeling of Lamb wave propagation in healthy composite plates through experimental measurements and conventional FEM;
3. Implemented 'baseline-free' Damage Force Indicator method for delamination detection using dynamic stiffness matrix from WSFE model of healthy plate and stiffened structures;
4. Developed Modified Time Reversal method for Lamb wave based 'baseline-free' damage diagnostics; and investigated effect of tone burst center frequency on instantaneous phase based delamination detection (This was accomplished by my Co-PI from USA, Dr Ratan Jha at Clarkson University)

All the objectives laid out in the proposal have been accomplished. The research resulted in significant visibility to the groups here at IISc and Dr Jha's group at Clarkson University. The research has resulted 3 journal papers and 9 conference papers so far and three articles are under preparation for journal publication.

1. SUMMARY OF ACCOMPLISHMENTS

Year 1[1] Accomplishment:

- Formulated Wavelet Spectral element for a healthy composite plates and used the formulated spectral element to obtain all the lamb wave modes. Validated the element with the conventional finite elements
- Formulated the wavelet spectral element for a plate with transverse cracks and validated this damaged element with conventional finite elements

[1] **Year 1 and Year 2 work was performed under contract Number FA 23861014086**

Year 2 Accomplishment:

- Development Wavelet Spectral elements for a composite plates with arbitrarily oriented cracks
- Development Laplace Transform based Spectral element formulation for plates with cracks
- Experimental Validation of health and Damaged spectral element. Experiments performed (at Clarkson University)

Year 3 Accomplishment:

- Developed Coupled FEM-Spectral Element Model for composite plates with arbitrary oriented cracks
- Experimentally validated the model (at Clarkson University)
- Formulated new damage measure to locate and quantify the extent of damage
- Experimentally validated the formulated damage measure (at Clarkson University)

Year 4 Accomplishment:

- Developed Spectral Finite element for stiffened aircraft structures. Two variants of elements developed one based on Plate-beam connections and the second based on Plate-Plate connections
- Numerically validated the developed spectral FEM through conventional FEM
- Extended the damage force method to predict debonds in stiffened structures.

TECHNICAL REPORT for Years 3-4

In this report, we will present the technical progress of the project for the years 3 and 4

ABSTARCT

In this work, the wave propagation analysis of built-up composite structures is performed using frequency domain spectral finite elements, to study the high frequency wave responses. The report discusses basically two methods for modeling stiffened structures. In the first method, the concept of assembly of 2D spectral plate elements developed and reported in the earlier report is used to model a built-up structure. In the second approach, spectral finite element method (SFEM) model is developed to model skin-stiffener structures, where the skin is considered as plate element and the stiffener as beam element. The SFEM model developed using the plate- beam coupling approach is then used to model wave propagation in a multiple stiffened structure and also extended to model the stiffened structures with different cross sections such as T-section, I section and hat section. A number of parametric studies are performed to capture the mode coupling, that is, the flexural-axial coupling present in the wave responses.

1. INTRODUCTION

The stiffened composite plates and the composite box-type structures are the building block of the wing sections of an aircraft and modeling wave propagation in these complex structures is still a challenging area of research. However, many methods are available, which accurately model the wave propagation in a simple rod, beam and plate type structures. Wave propagation problems, which are in the high frequency region, using the numerical methods such as finite difference method (FDM) (Strickwerda, 1989), the boundary element method (BEM)(Brebbia et al., 1984; Cho and Rose,1996), and finite element method (FEM) (Talbot and Przemieniecki, 1975; Koshiba et al., 1984; Zienkiewicz, 1989; Verdict et al., 1992; Yamawaki and Saito, 1992; Alleyne and Cawley, 1992) are computationally expensive and require a large computational memory, even in the case of 1D wave propagation problem. Basically, the FEM based methods require 15-30 nodes per shortest wavelength of the loading frequency to capture the structural wave parameters accurately (Schulte et al., 2010). The finite strip element method, which needs only a low level discretization has the problems due to the variable size of strip stiffness matrix and the requirement for the modifications of spline functions at boundary nodes (Cheung, 1976; Liu et al., 1990). Mass spring lattice model (MISLM), where lumped parameters are used to calculate inertia and stiffness properties (Delsanto and Mignogna, 1998; Yim and Sohn, 2000) and local interaction simulation approach (LISA), which is a heuristic approach (Delsanto et al.,1992; Delsanto et al.,1994; Delsanto et al.,1997), are some of the different approaches for modeling wave propagation, which are available in the literature. In all the above methods, proper distribution of mass matrix is a difficult step while modeling, in order to bring out accurately the wave characteristics of the structure. Various methods are available, which combines the accuracy of the spectral methods (Gottlieb and Orszag, 1977) and the flexibility of FEM. Among the spectral based methods, in the recent years, the fast Fourier transformation (FFT)-based spectral finite element method (SFEM), proposed by Doyle (1988) and the time

domain spectral element, proposed by Patera (1984), are the two methods, which are extensively used by the researchers, to model wave propagation in structures. In SEM, the use of, Lagrange polynomials at Gauss-Legendre-Lobbatto nodes or the Chebyshev polynomials at Chebyshev-Gauss-Lobatto points, makes the method advantageous for wave propagation problems, over the conventional FEM (Rucka, 2010). The major benefits are, the requirement of less number of nodes per shortest wavelength (10 or less, Rucka, 2010) and the diagonal mass matrix obtained by integrating the element matrices using the Gauss-Legendre-Lobbatto quadrature. In fact, SEM has been used extensively for solving wave propagation problems of simple structures using 1D (Kudela et al., 2007a), 2D (Zak et al., 2006; Kudela et al., 2007b) and 3D analysis (Peng et al., 2009; Kudela and Ostachowicz, 2009). On the other hand, in SFEM, the governing equation is solved exactly, in the frequency domain, which is used as interpolation function for element formulation and the inertial distribution of the structure is modeled, quite accurately. In the absence of any discontinuities, SFEM needs only one element to model a structure of any length. Further, SFEM is a fairly well established method and is extensively used to solve wave propagation problems in 1-Dwaveguides such as rods, beams or frames and in 2-D planar structures such as plates or membrane (Doyle, 1997; Mahapatra and Gopalakrishnan (2003); Chakraborty and Gopalakrishnan (2006); Gopalakrishnan et al., 2008). Monograph written by Gopalakrishnan et al. (2008) gives a complete overview of formulation of SFEM for 1-D and 2-D waveguides. The major drawbacks of SFEM include the problems due to the periodicity of the Fourier transform and the difficulty of obtaining the exact solutions of the transformed governing differential equations for every structure. However, the issues due to the periodicity of the Fourier transform can be avoided by using wavelet transform (Mitra and Gopalakrishnan, 2005) or Laplace transform (Igawa et al., 2004). Ham and Bathe (2012) proposed an enriched finite element method for 2D structures, which combines the advantages of finite element and spectral techniques by preserving the fundamental properties of FEM. The method does not embed 'a priori' specific wave solutions and hence it can be extended for solving complex problems, which involve material nonlinearities. In this method, harmonics to enrich the solution space can be selectively added, which makes it flexible and efficient for practical usage. In this work, we will use wavelet spectral element to model the built up composite sections using Debuanchis wavelets (See Mira and Gopalakrishnan, 2005 for the details of this method)

Modeling of stiffened and built-up structures is an order more complex compared to the planar structures. These structures can have plate-plate interfaces or plate-beam interfaces. A few works on modeling of stiffened structure using FEM for vibration analysis is reported in the literature such as Mukherjee and Mukhopadhyay (1988), Palani et al. (1992), Lee and Lee (1995), Kolli and Chandrashekhara (1996), Edward and Samer (2000), Gangadhara (2003) and Thinh and Khoa (2008) etc. However, modeling wave propagation in built-up structures is even more difficult and the literature available on wave propagation studies in these structures is minimal. Elastic wave propagation analysis in stiffened structure using analytical methods can be found in Fahy and Lindqvist (1976) and Grice and Pinnington (2000a,b). Some of the models for wave propagation in stiffened structure use the concept similar to homogenized model proposed by Timoshenko (1921) and Timoshenko and Woinowski-Krieger (1989). Gavric (1994) developed a numerical approach to model the cross section displacements using FEM, where the harmonically oscillating function is used to describe the displacement function. Orrennius and Finnveden

(1996) extended the Gavrric's original method (Gavric, 1994) to analyze the wave propagation in rib-stiffened plate by considering the built-up plate as an equivalent orthotropic plate, using a semi-analytical finite element technique, with improved computational efficiency. This work is limited to analyze the freely propagating waves. With the application of a load, the criteria for choice of an equivalent structure are more extensive, as the corresponding impedances must also be matched in addition to matching the wavenumbers of the propagating waves. Ichchou et al. (2008a,b) extended this work to the high frequency range using the concept of inhomogeneous wave correlation method (IWC). Satish Kumar and Mukhopadhyay (2002), developed a new stiffened plate element, which can accommodate any number of arbitrarily oriented stiffeners. Finnveden (2004) used an approach called waveguide-FEM to solve wave propagation problems of built-up thin walled structures. Lee et al. (2004) used higher order plate theory to investigate the dynamic behavior of multiply folded composite laminates. Mitra et al. (2004) developed a new super convergent thin walled composite beam element for wave propagation analysis of box beam structures. FEM based wave propagation model for I section can be found in Greve et al. (2005) and Aldrin et al. (2006) modeled Tsection geometry with fillets with and without notch defects. These FEM based models can model wave propagation in complex structures quite efficiently. However, they are computationally expensive and the modeling involves a crude error-bound approximation due to the numerical stability limit in computation (Gopalakrishnan et al., 2008). Some of the recent built-up structure models use SEM, due to its ability to accurately model complex structures and its computational efficiency, when compared to FEM. Rucka (2010) studied the longitudinal flexural wave propagation in a steel L-joint. Schulte et al. (2010), used the Gauss-Lobatto-Legendre (GLL) spectral element discretization based upon quadrangular elements for the wave propagation analysis of isotropic and anisotropic shell-structures and stiffened panels. Similarly, Schulte and Fritzen (2011) used SEM to study the propagation of waves in a curved panel with stiffeners. The model is computationally efficient and when compared to the FE, only five to six nodes (depending on the degree of the interpolation polynomial) per shortest wavelength of the excited frequency range are necessary to capture the structural behavior with the same accuracy as 15-30 nodes, which are needed using lower order FE. In the wave propagation analysis of 1D beam and 2D plate structures the requirement of 5e6 nodes per wavelength makes SEM computationally expensive, when compared to SFEM. In the area of SFEM based modeling of wave propagation in built-up structures, a wave propagation model for 3D frame structures by Doyle and Farris (1990) and a model by Danial et al. (1996), to analyze wave propagation in folded plate structures, are some of the earlier works reported in the archival of literature. However, these models are developed for isotropic structures. In this work, our aim is to develop a wavelet based wave propagation model for anisotropic stiffened structures of different cross sections, retaining the advantages of SFEM (in the case of 1D and 2D structures) such as computational efficiency, small system size and its ability to distribute the mass exactly, over the other methods, which are available in the literature. Further, Wavelet transform based damage models are efficient in detecting small scale damages in composite structures (Gopalakrishnan et al., 2008) and hence, in future the model can be extended as an efficient damage model for composite stiffened structures, with minimum system size. In the SFEM environment, modeling of stiffened and built-up composite structures, which involves plate-plate coupling, plate-beam coupling and different sections such as I, T and hat etc., is a novel concept and are quite

complicated and challenging. Modeling philosophy could be same as adopted in FEM, however these assembly in SFEM is different. Here, we first generate the dynamic stiffness matrices of each of the sub elements of composite construction, transform these dynamic stiffness matrices to global coordinates before assembling them. However, the method of coupling the structure with plate-plate interface and plate-beam interface are quite different. That is, spectral plate element involves solution that has double series summation to account for spatial and temporal modes, while the beam element solution involves only the summation of temporal modes. Hence, the couplings of structures involving plate-plate and plate-beam interfaces are treated separately. The use of SFEM based plate elements (Samaratanga et.al, 2013) and beam elements (Gopalakrishnan and Mira Mitra, 2011), which are computationally efficient, compared to the other methods such as FEM, SEM etc., makes the Wavelet transform-based SFEM model for stiffened structures, computationally efficient. Further, the fundamental aspects of modeling of built-up structures is that the presence of mode coupling. That is, an axial impulse creates flexural waves in the two planes of bending in a 3-D built-up structure. Keeping track of these multiple modes is indeed very challenging and this is will be addressed in future project.

The report is organized as follows. In the next section, a very brief description of the development of beam element and plate element, using SFEM is given, which is followed by the brief description of the method of modeling stiffened structures and box structures by assembling spectral plate elements. In the following section a method is developed to model stiffened structures using plate-beam assembly. The method of plate-beam assembly is then extended to model a stiffened structure, which contains T, hat and I sections. In the results section, the SFEM model using plate-plate assembly is first validated with 2D FE results and is then used to perform the high frequency wave propagation analysis of skin-stiffener structure and box structure. In the next sub-section, the method of modeling stiffened structures using plate-beam coupling is validated using 2D FE results and is used for modeling wave propagation in multiple stiffened structure. The plate-beam coupled SFEM model is then used to model high frequency wave propagation in stiffened structures with stiffeners of different cross sections such as T-section, I-section and hat section. A number of parametric studies are performed in each section, mainly to characterize the effect of flexural-axial coupling on the wave response.

2. WAVELET SPECTRAL ELEMENT FORMULATION

Wavelet spectral beam element formulation is reported in the monograph Gopalakrishnan and Mira Mitra (2011). This element is used to model the beam stiffener in the built up section. The details of the formulation are skipped here to keep the report short. The beam coordinate axis and the degrees of freedom is shown in Fig.1

The formulation begins with formulating governing differential equation, transforming to frequency domain using wavelet transform, performing uncoupling of wavelet coefficients using eigenvalue analysis, and writing the solution to the governing equation and using this solution to the spectral FE formulation. Finally, we will get the force-displacement relation through dynamic stiffness matrix as

$$\{\hat{F}\} = [\hat{K}]\{\hat{u}\} \qquad (1)$$

Here, $\{\hat{F}\}$ is the nodal force vector of size 6 x 1, $[\hat{K}]$ is the dynamic stiffness matrix of size 6 x 6 and $\{\hat{u}\}$ is the frequency domain displacement vector of size 6 x 1

Next we describe the formulation of the wavelet spectral plate element. This element is again formulated in Samaratanga et.al, 2013 and also reported in the earlier reports submitted by the PI.

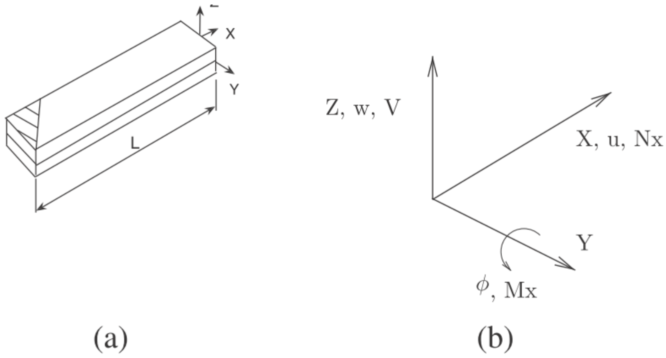

Figure 1: (a) Beam Geometry (b) Coordinate system and degrees of freedom

The displacement field unlike in beam is expressed as double summation as given below

$$\begin{Bmatrix} u(x,y,t) \\ v(x,y,t) \\ w(x,y,t) \\ \phi(x,y,t) \\ \psi(x,y,t) \end{Bmatrix} = \sum_{n=1}^{N} \sum_{m=1}^{M} \begin{Bmatrix} \widehat{u}(x) \\ \widehat{v}(x) \\ \widehat{w}(x) \\ \widehat{\phi}(x) \\ \widehat{\psi}(x) \end{Bmatrix} \begin{Bmatrix} \cos(\eta_m y) \\ \sin(\eta_m y) \end{Bmatrix} e^{-j\omega_n t} \quad \textbf{(2)}$$

In the above equation, vector $\{u \quad v \quad w \quad \phi \quad \psi\}^T$ denotes the three displacements in the 3 directions and two slopes are ϕ and ψ , respectively. In the above equation, η denotes the horizontal wavenumber. The plate geometry and the degrees of freedom are shown in Fig 2.

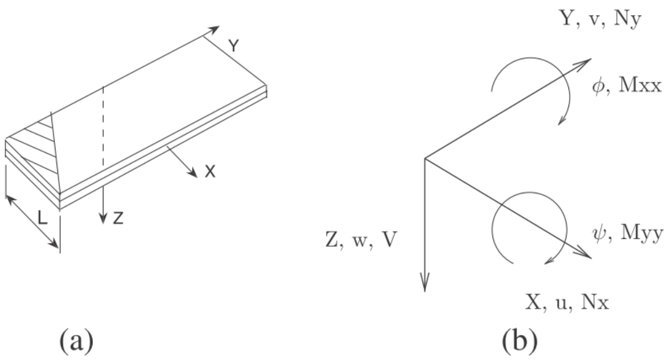

Figure 2: (a) Plate Geometry (b) Coordinate system and degrees of freedom

Following the same procedure as outlined for beam spectral element, this process will again yield a dynamic stiffness matrix $[\hat{K}(\omega_n,\eta_m)]$, which is dependent on both frequency ω and horizontal wavenumber η and the size of this matrix will be 10 x 10. Here, *n* and *m* are the frequency and horizontal wavenumber indices.

3. MODELING OF STIFFENED STRUCTURES USING PLATE ELEMENTS

A method of assembling 2D plate elements, which can be used for modeling complex structures, like stiffened structures and box structures (Figs. 3 and 4), is shown in Fig. 5. In stiffened structure, skin (element 1-2, Fig. 5) and the stiffener (element 2-3, Fig. 5) are modeled with plate elements.

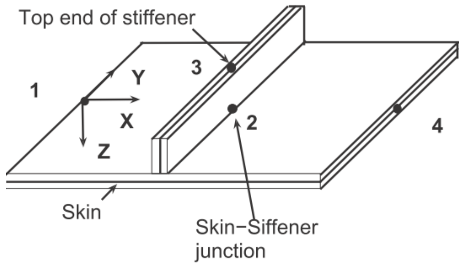

Figure 3: Schematic of a skin-stiffener structure.

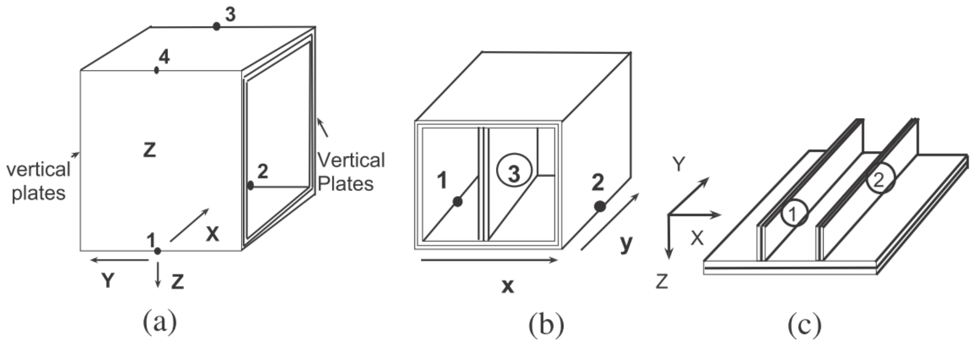

Figure 4: (a) Box structure (b) Box structure with two cells (c) Skin with two stiffeners.

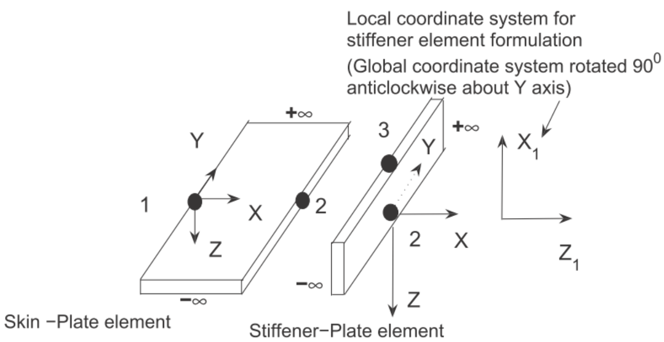

Figure 5: Plate-plate coupling

Stiffener is at an angle 90 degrees (anticlockwise) to the skin element 1-2 and hence, before assembling the elements 1-2 and 2-3, the stiffness matrix of the element 2-3, which is obtained in the local coordinate system ($X_1Y_1Z_1$, Fig. 5) is transformed in to global (XYZ, Fig. 5) coordinate system using a transformation matrix [T] of order 10 x 10 given by

$$\mathbf{T} = \begin{bmatrix} Q & 0 \\ 0 & Q \end{bmatrix}, \quad Q = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \tag{3}$$

$$[\mathrm{K_g}]_{n,m} = \left[\mathrm{T}^T\right][\mathrm{K}_l]_{n,m}[\mathrm{T}]$$

where $[K_l]_{n,m}$ is the element stiffness matrix in the local coordinate system ($X_1Y_1Z_1$, Fig. 5), $[K_g]_{n,m}$ is the transformed stiffness matrix in global coordinates (XYZ, Fig. 5) at each n and m and q is the rotation of the plate with respect to Y axis. In the next step, the stiffness matrix of element 1-2 and 2-3 can be assembled using a method, which is similar to the method of assembly, in conventional FEM.

Hence, the stiffened structure shown in Fig. 3 is modeled by assembling spectral plate elements, 1-2, 2-3, and 2-4 and the global stiffness matrix is of the order 20 x 20. The box structure, which is shown in Fig. 4(a) is modeled by assembling spectral plate elements, 1-2, 2-3, 3-4, and 4-1 (Global stiffness matrix is of order 20 x 20).

4. MODELING OF STIFFENED STRUCTURES USING 2D PLATE ELEMENTS AND 1-D BEAM ELEMENTS

In a stiffened structure analysis, usually the stiffeners are modeled as beam elements, especially, when they are thick sections. Hence, in this section, skin is modeled as spectral plate element and the stiffener is modeled as a Timoshenko beam element and they are coupled using a special procedure. The method of coupling the 2D spectral plate element with a 1D beam element is completely different from that of the method of assembling spectral plate elements (Section 3) since the plate elements solution involves summation of temporal and spatial modes (Eqn (2)) while the beam element has only summation of temporal modes. Summation of temporal mode is achieved using N number of frequency points, which is same for both the plates and beam elements. However, a plate element is also discretized spatially (in the Y direction), using Fourier series and is having M number of terms, as given in Eqn. (2). These M points are distributed evenly along the Y direction and at each of these points we need to couple the plate and beam stiffness matrices. Hence, in order to model a stiffener, which extends throughout the width (Y direction) of the skin structure (which is modeled using plate element), we need to assemble the stiffness matrix of the skin element (plate element) and the stiffener element (beam element) (for all N frequency values (ω_n), Eqn (2)), at each value of horizontal wavenumbers (η_m, Eqn(2)) used in the plate element formulation.

In the present study, stiffness matrix for the plate element is of order 10 x 10 and it varies with frequency (ω_n) and wavenumber (η_m) while the composite beam stiffness matrix is of the order 6 x 6 and varies only with frequency (ω_n). Nodal displacement vector of the beam element (element 2-3, Fig. 6) in the wavelet domain is $\{\hat{u}_2\hat{w}_2\hat{\phi}_2\hat{u}_3\hat{w}_3\hat{\phi}_3\}^T$ and the same for plate element (element 1-2, Fig. 6) is $\{\hat{u}_1\hat{v}_1\hat{w}_1\hat{\phi}_1\hat{\psi}_1\hat{u}_2\hat{v}_2\hat{w}_2\hat{\phi}_2\hat{\psi}_2\}^T$.

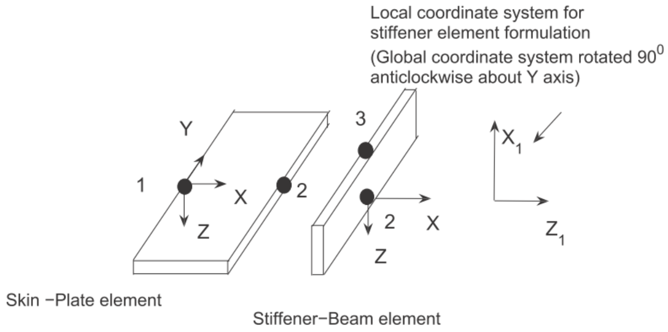

Figure 6: Plate-beam coupling.

In this study, the skin-stiffener structure, as shown in Fig. 3, is modeled by assuming spectral plate element model (Section 2) for the skin (1-2 and 2-4, Fig. 6) and a spectral beam element (Section 2) for the stiffener (element 2-3, Fig. 6) structure. Hence, in order to couple the plate element (1-2, Fig. 6) to the beam element (2-3, Fig. 6), at each value of the horizontal wavenumber (η_m), we should vary the value of frequency (ω_m) and add the corresponding dynamic stiffness matrix obtained for the beam element and plate element, which is given by

$$\begin{aligned}\Big(K_g n, m(ipt, ipt)_{\text{coupled}} &= \Big(\big(K_g(ipt, ipt)_n\big)_{\text{plate},1-2} \\ &\quad + \big(K_g(i,j)_n\big)_{\text{beam},2-3}\Big)\Big)_m \\ (i,j &= 1,..3, \quad ipt = [6,8,9])\end{aligned} \tag{4}$$

$$\begin{aligned}K_g n, m(ipt, ipt)_{\text{coupled}} &= \big(K_g(ipt, ipt)_{n,m}\big)_{\text{plate},1-2} \\ ipt &= [1,2,3,4,5,7,10]\end{aligned} \tag{5}$$

$$\begin{aligned}K_g n, m(ipt, ipt)_{\text{coupled}} &= \Big(\big(K_g(i,j)_n\big)_{\text{beam},2-3}\Big)_m, \\ i,j &= 4,..6, \quad ipt = [11,12,13]\end{aligned} \tag{6}$$

Here, the subscript *n* and *m* remind one that the coupled stiffness matrix is evaluated at a particular value of ω_n and η_m. The transformed coordinate system ($X_1Y_1Z_1$, Fig. 6) for the beam element formulation is shown in Fig. 6, which is obtained by rotating the global coordinate system (XYZ, Fig. 6) anticlockwise, by 90 degrees.The stiffness matrix for a vertical beam element (***K_{gbeam}***, Eqn. (6)) at each value of ω_n is obtained by transforming the stiffness matrix obtained in the local coordinates ($X_1Y_1Z_1$, Fig. 6) to global coordinates (XYZ, Fig. 6), in the same way, as we explained in Section 3. This is achieved by the transformation matrix T of order 6 x 6 given by

$$\begin{aligned}\mathbf{T} &= \begin{bmatrix} Q & 0 \\ 0 & Q \end{bmatrix}, \quad Q = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ \big[\mathrm{K_{gbeam}}\big]_n &= \big[\mathrm{T}^T\big][\mathrm{K}_l]_n[\mathrm{T}]\end{aligned} \tag{7}$$

where $[K_l]_n$ is the element stiffness matrix in local coordinates ($X_1Y_1Z_1$, Fig. 6, at each n), $[K_{gbeam}]_n$ is the transformed stiffness matrix in the global coordinates (XYZ, Fig. 6, at each n) and θ (90^0 Fig. 6) is the rotation of the plate with respect to Y-axis. A plate-beam coupled (coupling the elements,1-2 and 2-3 in Fig. 6) stiffness matrix is of the order 13 x 13 and finally while modeling a stiffened structure (Fig. 3), the global stiffness matrix is obtained by assembling the plate elements 1-2, 2-4 and the beam element 2-3 and this is of the order 18 x 18.

5. MODELING STIFFENED STRUCTURE WITH T-SECTION, I-SECTION OR HAT SECTION USING PLATE-BEAM COUPLING

In this section, we model wave propagation in skin-stiffener structures with stiffeners of various cross sections, which are shown in Fig. 7.

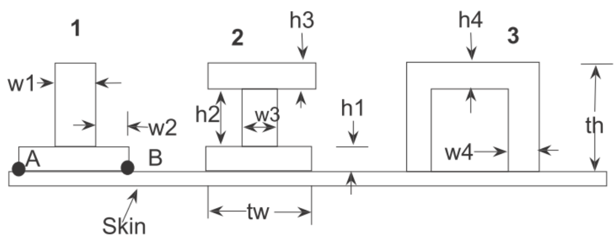

Figure. 7.: Different cross sections of the stiffeners considered for the study (1. T-section 2. I-section, and 3. Hat section.)

Here, the skin is modeled using spectral plate element and each part of the stiffener structure is modeled by spectral beam element (Section 2). However, before going to the analysis, we need to develop a method to couple plate and beam elements, where the beam is parallel to the plate and placed directly over the plate element, as shown in Fig. 8.

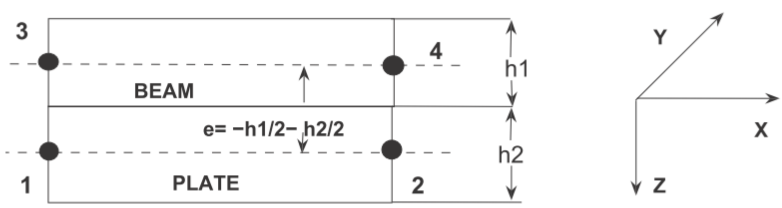

Figure. 8.: Schematic model of a beam placed directly over a plate element.

Here, a beam is attached over a plate, where the mid-planes of the plate and beam are parallel and the mid-plane of the beam is at a distance, '*e*' (Fig. 8) from that of the plate mid-plane and hence this offset in the mid-planes of the plate and beam is considered while coupling the elements. This coupled element can be modeled using a kinematic assumption for the interface of plate and beam, which is similar to the assumptions used in the references, Nag et al. (2003) and Thinh and Khoa (2008). The displacement compatibility between the stiffener and the plate is ensured by the beam elements displacement field, which is interpolated from plate elements nodes. The kinematic assumption for the interface of skin and stiffener is that the cross-section remains straight i.e. the slope is continuous and constant at the interface. Under this

assumption, one can obtain the following equations for the nodal displacements (Fig. 8)

$$\begin{Bmatrix} \widehat{u}_3 \\ \widehat{w}_3 \\ \widehat{\phi}_3 \end{Bmatrix}_{\text{beam}} = [S_1]\begin{Bmatrix} \widehat{u}_1 \\ \widehat{w}_1 \\ \widehat{\phi}_1 \end{Bmatrix}_{\text{plate}}, \quad [S_1] = \begin{bmatrix} 1 & 0 & e \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{Bmatrix} \widehat{u}_4 \\ \widehat{w}_4 \\ \widehat{\phi}_4 \end{Bmatrix}_{\text{beam}} = [S_1]\begin{Bmatrix} \widehat{u}_2 \\ \widehat{w}_2 \\ \widehat{\phi}_2 \end{Bmatrix}_{\text{plate}} \tag{8}$$

where the subscripts 1,2,3 and 4 are the corresponding nodes. Similarly total nodal forces at 1 and 2 can be written as

$$\begin{Bmatrix} \widehat{N}_x \\ \widehat{V}_x \\ \widehat{M}_x \end{Bmatrix}_i = \begin{Bmatrix} \widehat{N}_x \\ \widehat{V}_x \\ \widehat{M}_x \end{Bmatrix}_i + \begin{Bmatrix} \widehat{N}_x \\ \widehat{V}_x \\ \widehat{M}_x \end{Bmatrix}_j + \begin{Bmatrix} 0 \\ 0 \\ e\widehat{N}_x \end{Bmatrix}_j, \quad i = 1,2 \quad j = 3,4 \tag{9}$$

where, N_x, V_x and M_x are the axial force in X direction, shear force in Z direction and bending moment about Y axis, respectively. The subscript i and j denotes the node numbers of the plate and beam, respectively. Hence, the structure given in Fig. 8 can be modeled as a single element, where the nodal displacements of the beam can be expressed in terms of the nodal plate displacements (Eqn (8)) and the nodal forces can be expressed, as given in Eqn. (39). Element stiffness matrices for the plate and the beam can be obtained (explained in Section 2) and at each value of the horizontal wavenumber (η_m), we couple the plate stiffness matrix and the beam stiffness matrix (for all N frequency values), to derive the coupled plate-beam stiffness matrix. The plate-beam coupled stiffness matrix is of the order 10 x 10, and is obtained as follows:

$$\begin{aligned} \Big(K_g n, m(ipt, ipt)_{\text{coupled}} &= \Big(\big(K_g(ipt, ipt)_n\big)_{\text{plate, 1-2}} \\ &\quad + \big(K_g(i,j)_n\big)_{\text{beam, 3-4}}\Big)_m\Big) \\ (i,j = 1,..6, ipt &= [1,3,4,6,8,9]) \end{aligned} \tag{10}$$

$$\begin{aligned} K_g n, m(ipt, ipt)_{\text{coupled}} &= \big(K_g(ipt, ipt)_{n,m}\big)_{\text{plate,1-2}} \\ ipt &= [2,5,7,10] \end{aligned} \tag{11}$$

where $[K_{gplate}]$, is the stiffness matrix for the plate element 1-2 and $K_{\text{gbeam}} = K_{(\text{beam element 3-4, } 6\times 6)} \times \begin{bmatrix} S_1 & 0 \\ 0 & S_1 \end{bmatrix}_{6\times 6}$. Both the plate and beam stiffness matrix is obtained in XYZ coordinate system (global and local coordinates coincides), which is given in Fig. 8. Vertical beam element can be coupled by transforming the stiffness matrix in the local coordinate to global coordinates in the same way, which is explained in Section 4.

6. RESULTS AND DISCUSSION

6.1. Wave propagation in stiffened and built-up structure using plate-plate assembly:

In this section, the SFEM model, based on the spectral plate element assembly (Section 3), is used for obtaining the wave responses of a composite skin-stiffener structure (Fig. 3) and a box structure (Fig. 4(a)). Here, skin and stiffener of stiffened structure and all the faces of a box structure are considered as laminated composite plates and hence the whole structure can be modeled by the concept of assembling spectral plate elements, as explained in Section 3. The material used for the constituent laminated composite plates, is a GFRP composite, which has the following material properties: $E_1=144.48$ *Gpa*, $E_2=E_3=9.63$ *Gpa*, $G_{23}=G_{13}=G12=128GPa$, $\nu_{23}= 0.3$, $\nu_{13=}$ $\nu_{12}=0.02$ and $\rho_{12} = 1389\ kg/m^3$. In all the study, composite laminate considered consist of 8 layers and the thickness of each layer is assumed as 1 mm, unless specified otherwise. First, the model is validated by comparing the responses obtained from the SFEM model with that of the responses obtained using the 2D FE analysis. The model is then used, for a parametric study in stiffened structures, which includes the study of the effect of lay-up sequence, thickness and height of the stiffeners (element 2-3, Fig. 3), on the wave responses and also the model is applied to model multiple stiffened and multiple cell structures. The basic goal of this study is to extract the effect of flexural-axial coupling (due to the presence of stiffeners) in a skin-stiffener structure, on the wave responses. Transverse velocity responses obtained from the structures are mainly considered in the present study. However, some axial responses are also shown for comparing the responses with the transverse responses. In all the study, the responses are measured at the same point, where we apply the load.

6.1.1. Validation of SFEM model:

The SFEM model developed is first validated by comparing the transverse velocity responses of a stiffened structure and box structure obtained using the model with that of the responses obtained from 2D FE model. In SFEM, to model a skin stiffener structure with one stiffener (Fig. 3), three spectral plate elements are required, which results in a system matrix of order 20. In the case of a box structure (Fig. 4(a)), SFEM model needs only four spectral plate element, where the system matrix is of the same order 20. Skin-stiffener structure used for the study is 0.8 m in X- direction, 1.6 m in Y direction and a stiffener of 0.5 m height is attached at 0.6 m away (in X-direction) from node 1 (Fig. 3). Box structure extends to 1 m in X and Z directions and 2 m in Y direction. Material properties and other parameters of the laminate are same as we mentioned in the previous section (section 6). In SFEM, load is transformed to the frequency domain where 1024 sampling points are used. For spatial variation, 30 Fourier series coefficients are considered. In FE analysis, structure is modeled using 4-noded plate elements and to model the symmetric part of the skin-stiffener structure or the box structure, having geometric parameters as mentioned above, the analysis requires minimum 10,000 elements. While solving via FE analysis, Newmark's time integration method is adopted with a time increment of 1 μs. In the present study, in order to validate the accuracy of the plate-plate and plate-beam coupled models, we are interested mainly in the time of arrival of first reflection from the skin-stiffener junction (node 2, Fig. 3) and the top of the stiffener (node 3, Fig. 3). Hence, in order to maintain the consistency in the comparison, the length of the structure in the Y

direction is taken in such a manner that the reflection from the boundary will not reach with in the time frame of our interest. In this report basically we study the coupling between the symmetric longitudinal (axial) mode *u* (S_0 mode) and the anti-symmetric flexural mode *w* (A_0 mode). In this work, we use a broadband load (frequency content-70 kHz) and a tone-burst signal, modulated at 30 kHz, which is similar to the signals used by Gopalakrishnan et al. (2008). In each case, a transverse load is applied at a point and the response is measured at the same point. Both the structures are fixed at node 2.

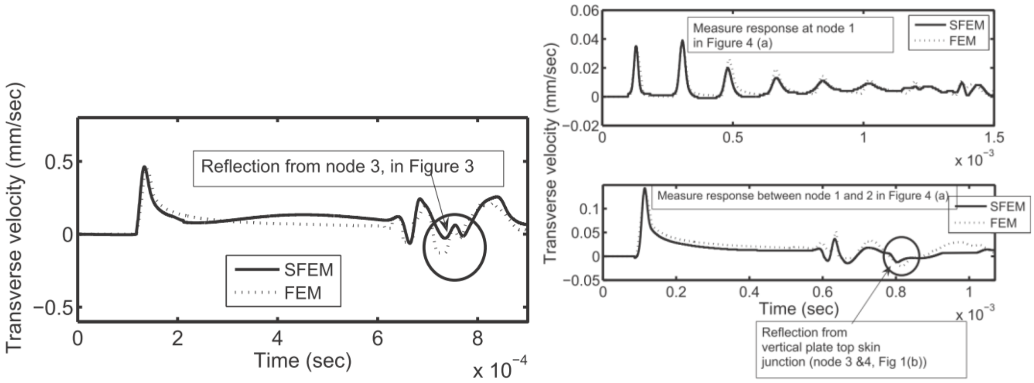

Figure 9: Transverse velocity response (a) skin-stiffener type structure (b) box structure.

Fig. 9(a) shows the transverse velocity response at node 1 (Fig. 3) of a skin-stiffener structure. Fig. 9(b) shows the transverse velocity responses at node 1 and at the midpoint of bottom skin (1-2, in Fig. 4(a)) of a box structure. Here, in all the plots, the waveform at 100 ms is the incident pulse. In Fig. 9(a), reflection (A_0 mode) from the junction of skin and stiffener starts (node 2, Fig. 3) at 0.62 ms and the reflection (S_0 mode) from the top free end of stiffener (node 3, Fig. 3), which is present due to the flexural-axial coupling, starts at 0.72 ms (marked in circle, Fig. 9(a)). Similarly, in the response at midpoint of the bottom skin (1-2, in Fig. 4(a)) of the box structure, reflection (A_0 mode) from the junction between the bottom skin and vertical plates (node 1 or 2, Fig. 4(a)) starts at 0.6 ms and the reflection (S_0 mode) from the top skin-vertical plate junction (node 3 or 4, in Fig. 4(a)) starts at 0.8 ms (Fig. 9(b)). In a box structure, for the response at node 1, the reflection (S_0 mode) from the junction between the top skin and vertical plate (node 4, in Fig. 4(a)) initiates at 0.3 ms and the reflection (A_0 mode) from the junction between the bottom skin and vertical plate (node 2, Fig. 4(a)) starts at 1.1 ms. In wave propagation analysis, the time gap between the incident pulse and the time for first reflection is a measure of the group speeds. Hence, if we know the distance traveled by the wave and the time gap between the incident pulse and reflected pulse, we can obtain the speed of the wave. Here, in the present study, the symmetric axial mode travels (at 10000 m/s) five times faster than the anti-symmetric flexural mode. The difference in the distribution of the mass of the structure causes a small difference in the amplitudes of the responses obtained using SFEM and FEM models. However, when we compare the results, there is an excellent match in the time of arrival of reflections, in the results obtained using both the models. In fact, the parameter, time of arrival of first reflection is the most important parameter, when we actually use the model for structural health monitoring applications. The wave propagation analysis of the structures mentioned

above, using FE analysis, needs large system size and consequently large computational time, when compared to the performance of the SFEM model. Here, in order to obtain the velocity responses for the stiffened structure, FEM model requires 115 min, while the SFEM model needs only 32 min (MATLAB code, Intel Core 2 Quad processor). The computational efficiency of SFEM depends on the total time window required to avoid the problem due to enforced periodicity and the time window can be adjusted by changing the time sampling rate or the number of FFT points. The increase in frequency further reduces the total time window needed for the analysis and consequently reduces the computational time (Ajith and Gopalakrishnan, 2011). However, in the conventional FEM, with the increase of frequency, the requirement of size of the element to be comparable with the wavelength makes the problem size so large that it becomes computationally prohibitive, especially in the high frequency range. Further, we can see that the effect of flexural-axial coupling due to the presence of stiffeners or the connecting plates, are well captured by the SFEM model.

6.1.2. Wave propagation in stiffened structures with stiffeners of rectangular cross section

In this section, we conduct a parametric study in a stiffened structure (Fig. 3). Here, skin and the stiffener are modeled as spectral plate element and these three plate elements are assembled, as mentioned in Section 3. The composite plates considered in this study consist of 8 layers and the material properties of each plate are same as the material properties, which we used in the validation section (Section 6.1.1). In the skin (1-2 in Fig. 3), each layer is 1 mm thick and the lay-up sequence is symmetric $[0]_8$. In most of the study, the load is applied in the transverse direction and the transverse responses are plotted. Axial responses are shown, only in a few cases, in order to compare the axial responses with that of the obtained transverse responses. The flexural-axial coupling in the wave response of a skin-stiffener structure due to the bonding of stiffeners on the skin needs to be analyzed. Skin is fixed at node 4 (Fig. 3). Load is applied at the free end of the structure (node 1, Fig. 3) and the responses are measured at the same point. The stiffener is placed in between the fixed and the free end of the structure (2-3, Fig. 3). Length of the skin structure in X and Y direction and the stiffener height are given inside the bracket while explaining each result. In all the case, the length of the structure in Y direction is taken larger compared to that of its length in X direction in order to avoid the effect of boundary reflections from the free end.

In wave propagation analysis, the real wavenumbers will only propagate. Here, we are interested in the study of flexural-axial coupling present in the wave responses of the stiffened structures. Hence, in the rest of the study, we choose either a broadband pulse of bandwidth 70 kHz or a tone-burst pulse, modulated at 30 kHz as the impact load. This is necessary to avoid the presence of shear modes (ϕ, ψ).

(a) Effect of height and thickness of the stiffener on the wave responses:

In the first case study, the effect of the height of the stiffener, on the transverse response is studied and the results are plotted in Figs. 10 and 11 (X =1 m, Y = 2 m and stiffener height varied from 0.5 m to 1.5 m). Even though the applied load is in the transverse direction, due to the flexural-axial coupling, S_0 mode is generated in the

stiffener and the trace of this mode can also be seen in the transverse velocity response of the skin-stiffener structure. The reflections (marked in circles, in Fig. 10) from the stiffener top (node 3, in Fig. 3) starts at 0.7 ms, in a 0.5 m high stiffener, where as in a 1.5 m high stiffener, the reflection will only start at 0.85 ms. The change in the time of arrival of first reflection from the stiffener with the change in the height of the stiffener can also be noticed in Fig. 12 (change from 0.7 ms to 0.85 ms), using the tone-burst signal. Change in thickness of the stiffener has different impact in the reflections (A_0 mode) from the skin-stiffener junction (node 2, in Fig. 3) and the reflections from the top end (node 3, Fig. 3) of the stiffener (S_0 mode). Increasing the thickness of the stiffener from 8 mm to 16 mm decreases the amplitude A_0 mode whereas it increases the S_0 mode (Fig.12(X=1m,Y= 2m and stiffener height = 0.5 m)).

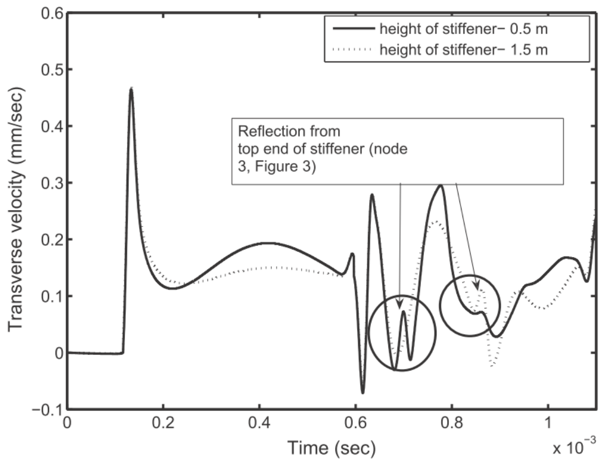

Figure 10: Transverse velocity response of skin-stiffener structure with different stiffener height, applying broadband load.

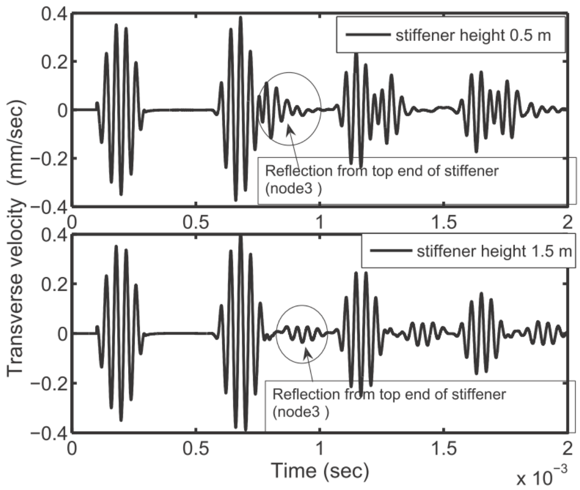

Figure 11: Transverse velocity response of skin-stiffener structure with different stiffener height, applying tone-burst load.

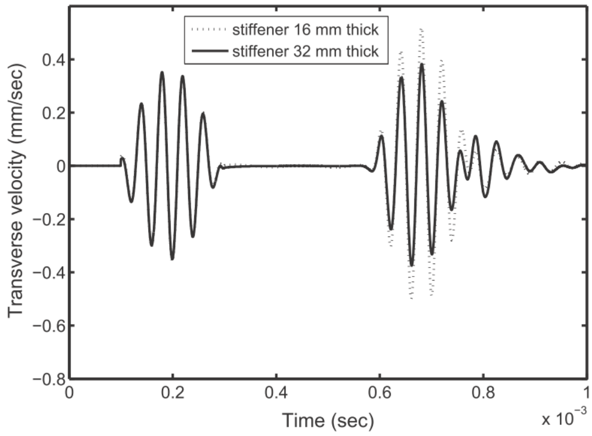

Figure 12: Transverse velocity response of skin-stiffener structure, varying the stiffener thickness, by applying tone-burst load.

(b) Axial and transverse wave responses:

The variation in the responses by changing the direction of the applied load from transverse to axial is shown in Fig.13 (X = 1 m, Y = 2 m and stiffener height = 0.5 m). The amplitude of the axial response is very small compared to the transverse responses. The trace of the reflections (A_0 mode) from the top of the stiffener can be noticed in the axial responses, which is due to the axial-flexural coupling. In axial responses, the effect of the change in length of stiffeners is not a significant factor, compared to the transverse responses (Fig. 13). Hence, in order to study the effect of bonding a stiffener in a skin structure, transverse responses are plotted in the rest of the study.

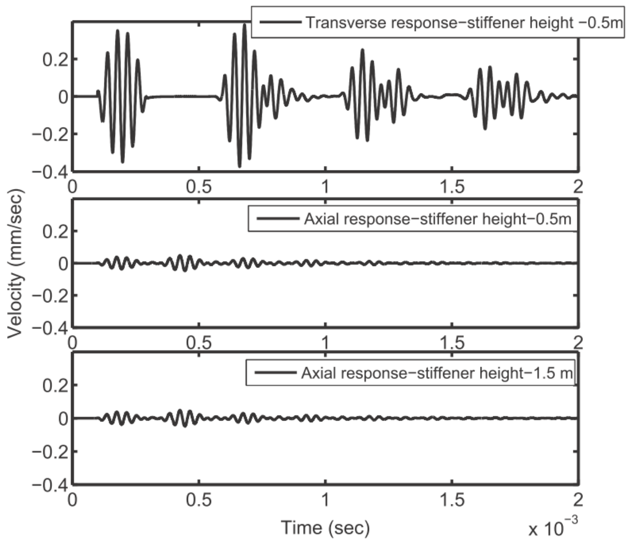

Figure 13: Transverse and axial response of skin-stiffener structure, applying tone-burst load.

(c) Wave propagation in a multiple stiffened structure:

Here, the number of stiffeners in a skin-stiffener structure is increased from one to two (Fig. 4(c)) and the response is obtained and is shown in Fig. 14. Skin is 2 m in X direction, 4 m in Y direction. In the case of single stiffener structure, stiffener is attached at 1 m away (in X direction, stiffener 2 in Fig. 4(c)) from the application of the load. In a double stiffener structure, in addition to the previous case, one more stiffener is attached at a point, 0.5 m from the point of application of the load (stiffener 1 in Fig. 4(c)). With the addition of a stiffener, additional reflections (both S_0 and A_0) are present (starts at 0.5 ms, in Fig. 14) in the wave responses. Hence, we can conclude that the SFEM plate-plate assembly model captures the effect of multiple stiffeners, quite efficiently.

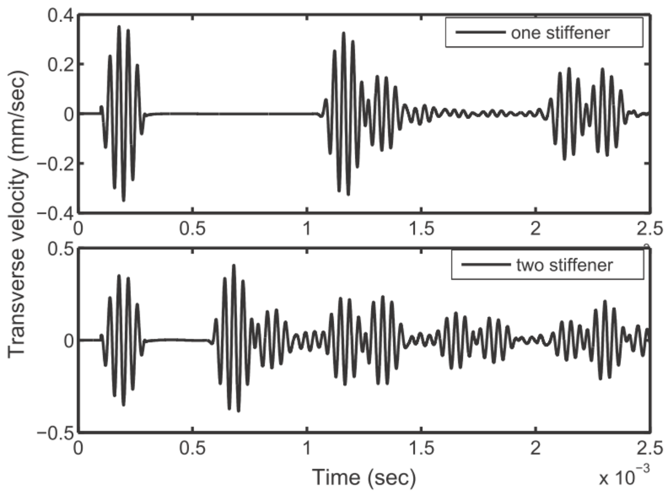

Figure 14: Transverse velocity response of skin-stiffener structure, varying the number of stiffeners, by applying tone-burst load.

6.2. Wave propagation in stiffened structure with stiffeners of rectangular cross sections using plate-beam assembly

In this sub-section, wave responses of a skin-stiffener structure are obtained using the SFEM model, which we discussed in Section 4. First, the model is validated by comparing the results obtained using the SFEM model with that of 2D FEM model, in Section 6.2.1, in the same way as we illustrated in Section 6.1.1. The model is then applied to perform the high frequency analysis of a multiple stiffened structure. In both the analysis, we use stiffeners of rectangular cross section (Fig. 3). The material properties and geometric properties of the skin and the stiffeners are same as that we used in the previous study using plate-plate assembly. Further, the load is applied at node 1 (Fig. 3), which is the free end and the responses are obtained at the same point, in the same manner as in Section 6.1.1.

6.2.1. Validation with 2D FE results.

Stiffened structure (Fig. 3) is modeled using two plate elements (1-2 and 2-4, Fig. 3), which represent the skin structure and a vertical beam element, which represent the stiffener (skin: X = 1 m, Y = 2 m and stiffener height = 0.5 m). Global stiffness matrix is of the order 18 x 18. The 2D FE analysis, performed here for the comparative study, is same as the analysis, which we used in Section 6.1.1. Fig. 15(a) shows a reasonable match between the SFEM model and the 2D FE model, which

shows the accuracy and the efficiency of the SFEM model. From the plot, it is observed that the SFEM model, which include the plate-beam coupling, captures both A_0 mode (at 0.6 ms, Fig. 15(a)) and S0 (at 0.7 ms, Fig. 18(a)) mode from the stiffener, accurately and with a small system size. Here, SFEM model (1024 frequency points) needs 30 min to perform the wave propagation analysis in the stiffened structure, while FEM requires 115 min.

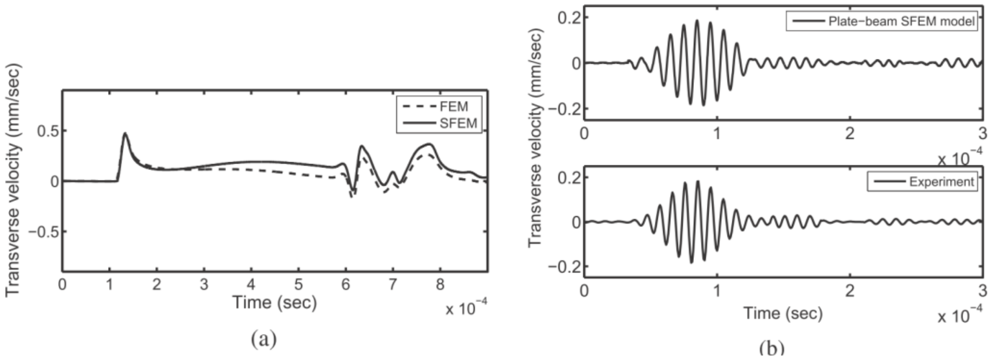

Figure 15: Validation of plate-beam SFEM model with (a) 2D FE analysis (b) Experimental results.

6.2.2. Validation with Experiments:.

In wave propagation modeling, plate-beam coupling is a new concept and hence to confirm the accuracy of the plate-beam model, we compare the results obtained from the model also with experimentally obtained results. In this section, transverse response of a stiffened structure (skin: X = 0.125 m, Y = 0.260 m and stiffener height = 0.075 m, stiffener located at the middle of the skin) obtained from experiment is compared with the response obtained from a plate-beam SFEM model. In SFEM, the structure is modeled in the same way as we did in the case of FE validation. Material properties of the skin and stiffeners are same as that we used in the previous section. A tone-burst signal modulated at 100 kHz is used as the load. Excitation is applied using Lead Zirconate Titanate (PZT) wafer active patch (15 mm diameter and 2 mm thickness) which was bonded to the structure. A 3D Laser Doppler Vibrometer (LDV) is used as the sensor to obtain the velocity response of the structure (Geetha Kolappan et al., 2012). LDVs measure velocities in the out of-plane direction by interferometrically measuring the change in frequency and phase of the back scattered laser light reflected from the surface. The result (Fig. 15(b)) shows that the reflections from the skin stiffener junction (A_0 mode) is well captured by the SFEM model and it matches quite well with the experimental results (reflections at 0.12 ms, 0.21 ms and 0.27 ms in Fig. 15(b)). In this study, the stiffener height is small and hence unlike the previous studies, it is difficult to distinguish the reflections from the top of the stiffener (S_0 mode, which is five times faster than A_0 mode), which is merged with the A_0 mode reflections from the skin-stiffener junction, in the velocity response.

7. CONCLUSIONS:

SFEM based models, are developed to study the wave propagation in box structures and stiffened structures, using the concept of assembling the existing 2D plate and 1D beam elements, which are spectrally formulated. The method of assembling the plate elements to model a stiffened structure or box structure is very simple and straight forward. However, we also developed a SFEM model, which can model wave

propagation in a plate-beam coupled stiffened structure. Both the SFEM models show excellent match with the 2D FE results. The accuracy of the plate-beam coupled model is confirmed by comparing the response obtained from the model with experimentally obtained response. Further, the models require only small system size and consequently less computational time, compared to 2D FE analysis while solving high frequency wave propagation problems of built-up composite structures. The effect of flexural-axial coupling on the wave responses is well captured using the SFEM models. The influence of geometric and the other laminate properties of the out-of plane components (stiffeners), which introduces the coupling, on the wave responses is studied, especially to characterize the change in the coupling due to the shift in these parameters. The wave propagation analysis of multiple cell box structures, multiple stiffened structures and also stiffeners of various cross sections (T, I and hat) is performed using the present SFEM models. The forward SFEM models thus developed can be used for the structural health monitoring applications of a general stiffened structure or box structure. The responses obtained for the healthy structures in the present study can be actually used to detect damages in a damaged composite stiffened structure, by comparing the responses obtained from the model with that of the experimentally obtained responses from the damaged structure.

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PUBLICATIONS FROM THIS PROJECT

Following Journal and conference papers came out of this project

1) **Samaratunga, D., Jha, R., and Gopalakrishnan, S., "**Wavelet Spectral Finite Element for Wave Propagation in Shear Deformable Laminated Composites Plates", ***Composites Structures,*** 108(1), 341-353-2014
2) **Gopalakrishnan, S.**, "Lamb wave Propagation in Laminated Composite Structures", ***Journal of Indian Institute of Science,*** 93 (4), 699-714, 2013
3) **Ajith V., and Gopalakrishnan, S., "**Wave Propagation in Stiffened Structures using Spectrally Formulated Finite Element", ***European Journal of Mechanics, A/Solids,*** **41,** 1-15, 2013
4) **Dulip Samaratunga, Jha, R., and Gopalakrishnan, S., "**Structural health monitoring of adhesively bonded composite joints using spectral finite element method", *Proceedings of Mechanics of Composites (MechComp 2014),* State University of New York, Stony Brook, New York, USA, June 8-12, 2014.
5) **Khalili, A., Dulip Samaratunga, Jha, R., and Gopalakrishnan, S.,** "Wavelet Spectral Finite Element Modeling of Laminated Composite Plates with Complex Features", *Proceedings of AIAA Science and Technology Forum and Exposition 2014, Paper no: 1740132,* National Harbor, Maryland, USA, January 13-17, 2014
6) **Dulip Samaratunga, Guan, X., Jha, R., and Gopalakrishnan, S., "**Wavelet spectral finite element method for modeling wave propagation in stiffened composite laminates", 54th *AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, Boston, Massachusetts, USA,* paper no*:* 1514286, April 8-11, 2013
7) **Gopalakrishnan, S., "**Structural Health Monitoring of Aerospace Structural Components Using Wave Propagation Based Diagnostics", *Proceedings of Sixth European Workshop on Structural Health Monitoring*, Vol 1, 26-34, Dresden, Germany, July 3-5, 2012 (Key Note Lecture)
8) **Gopalakrishnan, S., "** Wave Propagation in Built-up Aircraft Structures", *Proceedings of the Fourth International Conference on Computational Mechanics and Simulation* (ICCMS-2012), Indian Institute of Technology, Hyderabad, India, Dec 9-12, 2012
9) **Dulip Widana-Gamage, Ratneshawar Jha, and Gopalakrishnan, S., "**Wavelet Spectral Finite Element Modeling of Transverse Crack For Structural Health Monitoring Of Composite Plates", Paper No: AIAA-2012-1219314, *53rd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference,* Honolulu, Hawaii, April 23-26, 2012
10) **Ajith, V., and Gopalakrishnan, S., "**Guided-Wave Based Structural Health Monitoring of Built-Up Composite Structures Using Spectral Finite Element Method ", Paper Number 8348-22, *SPIE Structures/NDE Conference***,** San Diego, California, USA, March 11-15, 2012
11) Gopalakrishnan, S., and Jha, R., " A Wavelet Spectral Element for Composite Plate with Delamination and Transverse Damage", Paper No:AIAA-2010-2901,51stAIAA/ASME/ASCE/AHS/ASCStructures,

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12) Gopalakrishnan, S., and Jha, R. Inho Kim, and Dulip Widana-Gamage ., “Composite Delamination Detection Using Wavelet Spectral Finite Element and Damage Force Indicator Method”, 2901,51stAIAA/ASME/ASCE/AHS/ASCStructures, Structural Dynamics, and Materials Conference, Denver, Colorado, April4-7, 2011